1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part1, Class XI)
Module Name/Title	Unit 1, Module 4, **Dimensional Analysis** Chapter 2, Units and Measurements
Module Id	Keph_10203_eContent
Pre-requisites	Students should have knowledge of fundamental and derived quantities. SI units, basic algebra ,exponents and powers
Objectives	**After going through this module, the learners will be able to**: • Understand dimensions of a physical quantity • Derive the dimensional formula of recognized physical quantities • Apply method of dimensional analysis for deriving and checking formulae and equations • Recognize the Limitations of dimensional analysis
Keywords	Dimensions, dimensional formula, principle of homogeneity, applications of dimensional analysis, limitations of dimensional analysis

2. Development Team

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1. UNIT SYLLABUS

Unit 1: PHYSICAL WORLD AND MEASUREMENT

Chapter-1: Physical world

Physics- scope and excitement; Nature of physical laws; Physics, technology and society.

Chapter-2: Units and Measurements

Need for measurement: Units of measurement; Systems of units; SI units, fundamental and derived units. Length, Mass and Time measurements; Accuracy and Precision of measuring instruments; errors in measurements; significant figures.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 4 Modules

The unit is divided into four modules for better understanding.

Module 1	• Physical world • Meaning of physics. • Scope and Excitement of physics.
Module 2	• Need of measurement • SI units • fundamental and derived units • Measurement of mass ,length and time
Module 3	• Accuracy, precision • significant figures • Errors
Module 4	• Expressing physical quantities dimensionally • Dimensional analysis • Application of dimensional analysis

Module 4

3. WORDS YOU MUST KNOW

- **Fundamental units**: These units can neither be derived from one another nor can be resolved into any other units. They are independent of one another. Units of mass, length and time are fundamental units.

- **Derived units**: All other Units of physical quantities which can be expressed in terms of fundamental units are called derived units.

- **System of units**: A complete set of units which is used for measuring all kinds of fundamental and derived quantities is called a system of units.

- **Exponent** Exponentiation is a mathematical operation, written as y^a, involving two numbers, the base x and the exponent or power a. When a is a positive integer, it means repeated multiplication of the base: that is, y , a times .

$$\boldsymbol{y^3 = y \times y \times y}$$

If exponent or power is negative $$\mathbf{y^{-3} = \frac{1}{y^3} = \frac{1}{y} \times \frac{1}{y} \times \frac{1}{y}}$$

- **Rules for exponents**

1) Zero Property of Exponent

$$b^0 = 1$$

2) Negative Property of Exponent

$$b^{-n} = \frac{1}{b^n} \quad \text{OR} \quad \frac{1}{b^{-n}} = b^n$$

3) Product Property of Exponent

$$(b^m)(b^n) = b^{m+n}$$

4) Quotient Property of Exponent

$$\frac{b^m}{b^n} = b^{m-n}$$

5) Power of a Power Property of Exponent

$$(b^m)^n = b^{mn}$$

6) Power of a Product Property of Exponent

$$(ab)^m = a^m b^m$$

7) Power of a Quotient Property of Exponent

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

4. INTRODUCTION

While doing physics problems, we are often required to determine the numerical value and the units of a physical quantity, constant or variable in an equation. The numerical value usually is not too difficult to get, but for a beginner, the same cannot be said for the units.

In this module we will cover dimensional analysis which is a useful method for determining the units of a variable in an equation. We will also learn that dimensional

analysis can also be used to check the dimensional consistency of equations, and also for finding relations among various physical quantities.

5. DIMENSIONS, DIMENSIONAL FORMULA AND DIMENSIONAL EQUATION

We have learnt about fundamental and derived quantities in the previous modules .The physical quantities which are called as derived quantities can be expressed in terms of a combination of the seven fundamental or base quantities. We shall call these base quantities as the seven dimensions of the physical world. These fundamental quantities are denoted with square brackets [ ]. Using a square brackets [ ] around a physical quantity means that we are dealing with the dimensions of the quantity.

The seven dimensions of the physical world are represented as follows:

[L] for length

[M] for Mass

[T] for Time

[A] for Electric current

[K] for Temperature

[Cd] for Luminous intensity

[mol] for Amount of substance

Whenever a particular derived quantity is expressed in terms of fundamental or base quantities, it is written as a product of different powers of the fundamental quantities and the powers to which fundamental quantities must be raised in order to express the given physical quantity completely are called its **dimensions.**

FOR EXAMPLE:

Consider the physical quantity, momentum.

Momentum is defined as product of mass and velocity(we will try to consider physical quantities with which you are familiar from your earlier courses in science)

Momentum = mass x velocity

$$= \text{mass} \times \frac{\text{distance travelled}}{\text{time}}$$

$$= \text{mass x length x (time)}^{-1}$$

Here distance travelled is measured as length

We write it in a special way, square brackets and exponents to indicate how mass [M], length[L] and time[T] are involved in getting the physical quantity

$$= [M]\,[L]\,[T^{-1}]$$

$$= [M^{1}L^{1}T^{-1}].$$

Here, powers of fundamental quantities mass, length and time are 1, 1,-1 respectively.

Thus, the **dimensions** of momentum are 1 in mass, 1 in length and –1 in time.

DIMENSIONAL FORMULA

The expression which shows how and which of the quantities represent the dimensions of a particular physical quantity is called the dimensional formula of the given physical quantity.

FOR EXAMPLE:

The dimensional formula of the given physical quantity, momentum is $[M^{1}L^{1}T^{-1}]$.

DIMENSIONAL EQUATION

An equation obtained by equating a physical quantity with its dimensional formula is called **dimensional equation of** the physical quantity.

FOR EXAMPLE:

[Momentum] = $[M^{1}L^{1}T^{-1}]$

Such an expression for a physical quantity in terms of the fundamental quantities is called the **dimensional equation**.

Let's take some more examples

i). Area = length x breadth

$= [L]\,[L]$

$= [L^{2}]$

$= [M^{0}L^{2}T^{0}]$

Hence the dimensions of area are, 0 in mass, 2 in length, and 0 in time.

Dimensional formula of area is $[M^{0}L^{2}T^{0}]$

Dimensional equation of area is, [area] = $[M^{0}L^{2}T^{0}]$

ii) Volume = length x breadth x height

$= [L]\,[L]\,[L]$

$= [L^{3}]$

$= [M^{0}\,L^{3}\,T^{0}]$

Thus, the dimensions of volume are, 0 in mass, 3 in length, and 0 in time.

Dimensional formula of volume is $[M^{0}\,L^{3}\,T^{0}]$

Dimensional equation of volume is [Volume] = $[M^{0}\,L^{3}\,T^{0}]$

iii). Speed = Distance/time

= length/time

$= [L]\,[T^{-1}]$

$= [M^0\,L^1T^{-1}]$

Thus, the dimensions of speed are, 0 in mass, 1 in length, and -1 in time.

Dimensional formula of speed is $[M^0\,L^1T^{-1}]$

Dimensional equation of speed is [Speed] = $[M^0\,L^1T^{-1}]$

iv) Force = mass × acceleration

acceleration is rate of change of velocity or speed

Force = mass$\times \frac{\boldsymbol{speed}}{\boldsymbol{time}}$

But speed = $\frac{\boldsymbol{length}}{\boldsymbol{time}}$

Force = mass$\times \frac{\boldsymbol{length}}{\boldsymbol{time}} \times \frac{\boldsymbol{1}}{\boldsymbol{time}}$

Using the exponent rule we can say

force = mass × length × $(\text{time})^{-2}$

$= [M^1\,L^1\,T^{-2}]$

Thus, the dimensions of force are, 1 in mass, 1 in length and – 2 in time.

Dimensional formula of force is = $[M^1\,L^1\,T^{-2}]$

Dimensional equation of force is [force] = $[M^1\,L^1\,T^{-2}]$

As discussed above you can find the dimensions of any given physical quantities.

The following link is about dimensions.

https://youtu.be/Xb-muslnwPM

DIMENSIONLESS QUANTITIES

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There are some quantities which have no dimensions.

For example,

The sine of an angle i.e. sin θ is defined as the ratio of the lengths of two particular sides of a right angled triangle.

Thus, the dimensional formula of the sin θ is

[L]/ [L],

= $[M^0L^0T^0]$

Therefore, the sine function is said to be **"dimensionless"**. There are many other examples of "dimensionless" quantities listed below,

I. **All trigonometric functions**
II. **Exponential functions**
III. **Logarithms**
IV. **Angles**
V. **Quantities which are simply counted, such as the number of students in the classroom, vehicles on the road, stars in the galaxies etc.**
VI. **Plain numbers (like 2, π, etc.)**

6. DIMENSIONAL FORMULAE AND S.I. UNITS OF SOME PHYSICAL QUANTITIES

You have learnt about some of the physical quantities in your previous classes, and you will learn about some of them later, in class 11.

S.NO.	PHYSICAL QUANTITY	FORMULA	SI. UNIT	DIMENSINAL FORMULA
1.	Area(A)	length × breadth	m^2	$[M^0L^2T^0]$
2.	Volume (v)	length × breadth x height	m^3	$[M^0 L^3T^0]$

3.	Velocity or speed (v)	Distance/time	m/s	$[M^0L^1T^{-1}]$
4	Acceleration (a)	Change in velocity/time	m/s^2	$[M^0L^1T^{-2}]$
5	Momentum (P)	Mass × velocity	kg-m/s	$[M^1L^1T^{-1}]$
6.	Density (d)	Mass/volume	kg/m^3	$[M^1L^{-3}T^0]$
7	Force (F)	Mass × acceleration	Newton	$[M^1L^1T^{-2}]$
8	Impulse (I)	Force × time	Newton-sec or kg-m/s	$[M^1L^1T^{-1}]$
9	Pressure (P)	Force/area	Pascal	$[M^1L^{-1}T^{-2}]$
10	Kinetic energy (E_K)	½ (mv^2)	Joule	$[M^1L^2T^{-2}]$
11	Work (W)	Force × distance	Joule	$[M^1L^2T^{-2}]$
12	Power (P)	Work/time	Watt or Joule/s	$[M^1L^2T^{-3}]$
13	Trigonometric ratios sinθ, cosθ, tanθ etc.	Length/length		$[M^0L^0T^0]$
14	Angle, Angular displacement (θ)	Arc/radius	Radian (rad.)	$[M^0L^0T^0]$
15	Angular velocity (ω)	Angular displacement/time	Radian/sec	$[M^0L^0T^{-1}]$
16	Angular acceleration (α)	Angular velocity/time	$Radian/sec^2$	$[M^0L^0T^{-2}]$
17	Moment of inertia (I)	Mass × $(distance)^2$	$kg\text{-}m^2$	$[M^1L^2T^0]$
18	Torque (τ)	Force × distance	Nm	$[M^1L^2T^{-2}]$

19	Angular momentum (L)	Mass × velocity × radius	Joule-sec	$[M^1L^2T^{-1}]$
20	Force constant or spring constant (k)	Force/displacement	Newton/m	$[M^1L^0T^{-2}]$
21	Gravitational constant (G)	(Force × (distance)^2)/ (mass mass)	N-m^2/kg^2	$[M^{-1}L^3T^{-2}]$
22.	Pressure gradient	Pressure/distance	Pa m^{-1}	$[M^1L^{-2}T^{-2}]$
23.	Radius of gyration	Distance	m	$[M^0 L^2T^0]$
24	Surface tension (T)	Force/length	N/m	$[M^1L^0T^{-2}]$
25	Velocity gradient	Velocity/distance	Second^{-1}	$[M^0L^0T^{-1}]$
26	Coefficient of viscosity (η)	Force × distance/area × velocity	kg/m-s	$[M^1L^{-1}T^{-1}]$
27	Stress	Force/area	N/m^2	$[M^1L^{-1}T^{-2}]$
28	Strain	Change in configuration /original configuration	No unit	$[M^0L^0T^0]$
29	Modulus of elasticity (E)	Stress/strain	N/m^2	$[M^1L^{-1}T^{-2}]$
30.	wavelength	Distance	m	$[M^0L^1T^0]$
30	Time period (T)	Time	Second	$[M^0L^0T^1]$
31	Frequency (υ)	1/time period	Hz	$[M^0L^0T^{-1}]$
32	Temperature (T)		Kelvin	$[M^0L^0T^0 K^1]$
33	Heat (Q)	Energy	Joule	$[ML^2T^{-2}]$
34	Specific Heat (c)	Heat/mass × temperature	Joule/kg K	$[M^0L^2T^{-2} K^{-1}]$
35	Heat capacity, entropy	Heat /temp	Joule/K	$[M^1L^2T^{-2} K^{-1}]$

36	Latent heat (L)	Heat/mass	Joule/kg	$[M^0L^2T^{-2}]$
37	Gas constant (R)	PV/nT	Joule/mol*K	$[M^1L^2T^{-2}K^{-1}]$
38	Boltzmann constant (k)	Energy/temperature	Joule/K	$[M^1L^2T^{-2}K^{-1}]$
39	Coefficient of thermal conductivity (K)	Heat × distance/area × temp × time	Joule/m-s-K	$[M^1L^1T^{-3}K^{-1}]$
40	Stefan's constant (σ)	Energy/area × time × $(\text{temp})^4$	Watt/m^2-K^4	$[M^1L^0T^{-3}K^{-4}]$
41	Wien's constant (b)	Wavelength × temperature	Meter-K	$[M^0L^1T^oK^1]$
42	Planck's constant (h)	Energy/frequency	Joule-s	$[M^1L^2T^{-1}]$
43	Coefficient of thermal Expansion	Change in dimension/original dimension x temp	Kelvin^{-1}	$[M^0L^0T^0K^{-1}]$
44	Mechanical eq. of Heat (J)	J=W/H	Joule/Calorie	$[M^0L^0T^0]$
45.	Refractive index(μ)	Speed of light in vacuum/speed of light in medium	No unit	$[M^0L^0T^0]$
46	Electric charge (q)	Current× time	Coulomb	$[M^0L^0T^1A^1]$
47	Electric current (I)		Ampere	$[M^0L^0T^0A^1]$
48	Electric potential(V)	Work/charge	volt	$[M^1L^2T^{-3}A^{-1}]$
49	Resistance (R)	Pot.diff/current	ohm	$[M^1L^2T^{-3}A^{-2}]$
50	Resistivity or Specific resistance (ρ)	RA/l	Ohm-meter	$[M^1L^3T^{-3}A^{-2}]$

On observing the dimensional formulae of the quantities above, we find that some quantities share the same dimensional formula. Following is the list of those quantities:

S. N.	**Dimensional formula**	**Physical quantities**
1.	$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity gradient
2.	$[M^1L^2T^{-2}]$	Work, , potential energy, kinetic energy, torque, heat
3.	$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, modulus of elasticity.
4.	$[M^1L^1T^{-1}]$	Momentum, impulse
5.	$[M^0L^1T^{-2}]$	Acceleration, Acceleration due to gravity, gravitational field intensity
6.	$[M^1L^1T^{-2}]$	force, weight,
7.	$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
8.	$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area)
9.	$[M^0L^0T^0]$	Strain, refractive index, angle.
10.	$[M^0L^2T^{-2}]$	Latent heat and gravitational potential

This video provides a basic overview of Dimensional Analysis as well as some examples of how it is used in Physics.

https://youtu.be/c_ZUnEUlTbM

https://youtu.be/BRKI13Wb_T4

7. TYPES OF PHYSICAL QUANTITIES

1	Dimensional variables	The physical quantities which possess dimensions and variables values. **For example,** speed ,force, momentum etc.
2	Dimensionless variable	The physical quantities which do not possess dimensions but have variables values. **For example** , angle, strain etc.
3	Dimensional constants	The physical quantities which possess dimensions but have constant values **For exemple:** Gravitationnel constant, Plancks constant etc.
4	Dimensionless constants	The physical quantities which do not possess dimensions and have constant values **For example**: e ,π etc

8.APPLICATIONS OF DIMENSIONAL ANALYSIS

The process of studying a physical phenomenon on the basis of dimensions is called dimensional analysis. The dimensional formulae help us to understand the physical behavior of a quantity. Two physical quantities having different dimensions cannot be added or subtracted that means we cannot add area to volume, velocity to acceleration or subtract force from time. Thus we must remember that:-

1. If X + Y and X – Y are meaningful then, X and Y have same dimensions.

2. If X=Y is correct then X and Y have same dimensions and same nature.

This concept, i.e., only like quantities can be added or subtracted, can be applied to check the correctness of a mathematical equation.

This concept is applied in the **three main applications of dimensional analysis** which are:

i). **To convert one system of unit into another.**

ii) To check the accuracy of the formula

iii) To derive the formula

We will discuss these applications one by one.

(i)CONVERSION OF UNITS

As we have discussed in previous module that the magnitude of a physical quantity remains the same irrespective of the system of unit chosen to express the measurement.

The numerical value of a physical quantity in a system of units can be changed to another system of units using the equation nu = constant, where n is the numerical value and u is the dimensional value in that system of unit.

The measure of a physical quantity i.e.

Q = nu = constant

$n_1 u_1 = n_2 u_2$

Let a physical quantity z has dimensional formula$[M^a\, L^b\, T^c]$ and if (derived) units of that physical quantity in two systems are $[M_1^a\, L_1^b\, T_1^c]$ and $[M_2^a\, L_2^b\, T_2^c]$ respectively and n_1 and n_2 be the numerical values in the two systems respectively, then

$$n_1[u_1] = n_2[u_2]$$

$$n_1[M^a{}_1 L^b{}_1 T^c{}_1] = n_2[M^a{}_2 L^b{}_2 T^c{}_2]$$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

Where M_1, L_1 and T_1 are fundamental units of mass, length and time in the first (known) system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the second (unknown) system. Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

The following video is about application of dimensional analysis

https://youtu.be/xQs_3ZQ_o6k

Let's consider few examples to understand the application of the above formula.

EXAMPLE

Convert 1 Newton in to Dynes.

SOLUTION :

We know that, Newton is the S.I. unit (first system) and dynes is the CGS unit (second system) of force and dimensional formula of force is $[M^1L^1T^{-2}]$

$a = 1, b = 1, c = -2$

First system (SI system)

$M_1 = 1Kg$

$L_1 = 1\ m$

$T_1 = 1\ sec$

$n_1 = 1$

Second system (CGS system)

$M_2 = 1g$

$L_2 = 1$ cm

$T_2 = 1$ sec

n_2 = to be calculated

By using:

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$n_2 = 1 \left[\frac{kg}{gm}\right]^1 \left[\frac{m}{cm}\right]^1 \left[\frac{sec}{sec}\right]^{-2}$$

$$n_2 = 1 \left[\frac{10^3\,gm}{gm}\right]^1 \left[\frac{10^2 cm}{cm}\right]^1 \left[\frac{sec}{sec}\right]^{-2}$$

$$n_2 = 10^5$$

∴ 1 N = 10^5 Dyne

EXAMPLE

Convert 10 joule into ergs.

SOLUTION:

10 joule = ?ergs.

$n_1 u_1 = n_2 u_2$

We know that, joule is the S.I. system of unit (first system) and ergs is the CGS system of unit (second system) of energy and dimensional formula of energy is $[M^1L^2T^{-2}]$

$a = 1, b = 2, c = -2$

First system (SI system)

$M_1 = 1$Kg

$L_1 = 1$ m

$T_1 = 1$ sec

Second system (C G S system)

$M_2 = 1g$

$L_2 = 1$ cm

$T_2 = 1$ sec

n_2 = to be calculated

By using

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$n_2 = 10\,[1\text{kg}/1\text{gm}]^1\,[1\text{m}/1\text{cm}]^2\,[1\text{sec}/1\text{sec}]^{-2}$

$= 10 \times 1000 \times 100 \times 100 \times 1$

$= 10^8$

That means 10 joule = 10^8 ergs.

EXAMPLE

In C.G.S. system the magnitude of the force is 100 dynes. Find the magnitude of the force in another system which uses the fundamental physical quantities as kilogram, meter and minute.

SOLUTION:

We know the dimensional formula of force is $[M^1L^1T^{-2}]$

So $a = 1, b = 1, c = -2$

First system (C.G.S. system**)**

$M_1 = 1$gm

L_1 = 1cm

T_1 = 1 sec

Second system (new system)

M_2 = 1kg

L_2 = 1 m

T_2 = 1 minute = 60 sec

n_2 = to be calculated

By substituting these values in the following conversion formula

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$n_2 = 100 \left[\frac{gm}{kg}\right]^1 \left[\frac{cm}{m}\right]^1 \left[\frac{sec}{minute}\right]^{-2}$$

$$n_2 = 100 \left[\frac{gm}{10^3 gm}\right]^1 \left[\frac{cm}{10^2 cm}\right]^1 \left[\frac{sec}{60 sec}\right]^{-2}$$

$$n_2 = 100 \times 10^{-3} \times 10^{-2} \times 60 \text{ x } 60$$

$$n_2 = 3.6$$

100 dynes = 3.6 in new system of units.

The following link is for the video on dimensional analysis.

https://youtu.be/idmKjD78OHk

(ii) TO CHECK THE DIMENSIONAL CONSISTENCY OF A GIVEN PHYSICAL RELATION:

This application is based on the '*principle of homogeneity*'. According to this principle, the dimensions of each term on both sides of an equation must be the same.

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not.

EXAMPLE

Check the accuracy of the following relation.

$$\boldsymbol{F = \frac{mv^2}{r^2}}$$

SOLUTION:

By substituting dimension of the physical quantities in the above relation –

$[\mathrm{M\ L\ T^{-2}}] = \frac{[\mathrm{M}]\,[\mathrm{LT^{-1}}]^2}{[\mathrm{L}]^2}$

i.e.,

$[\mathrm{M\ L\ T^{-2}}] = [\mathrm{MT^{-2}}]$

As in the above equation dimensions of both sides are not same; this formula is not correct dimensionally.

EXAMPLE

Check the accuracy of the following relation.

$S = u\,t + 1/2\ at^2$

SOLUTION:

We will substitute the dimensions of the physical quantities in the above relation –

Remember Each term on both sides of the equation must have the same dimensions

$[\mathrm{L}] = [\mathrm{LT^{-1}}]\,[\mathrm{T}] + [\mathrm{LT^{-2}}]\,[\mathrm{T^2}]$

i.e. $[\mathrm{L}] = [\mathrm{L}] + [\mathrm{L}]$

$[\mathrm{L}] = [\mathrm{L}]$

As in the above equation dimensions of each term on both sides are same, so this equation is dimensionally correct.

(iii) DEDUCING RELATIONSHIP AMONG VARIOUS PHYSICAL QUANTITIES

The concept of dimensions can be used to find how various physical quantities depend on each other.

If we know the dependency of a physical quantity on other quantities and if the dependency is of the product type, then using the method of dimensional analysis, relation between the quantities can be derived.

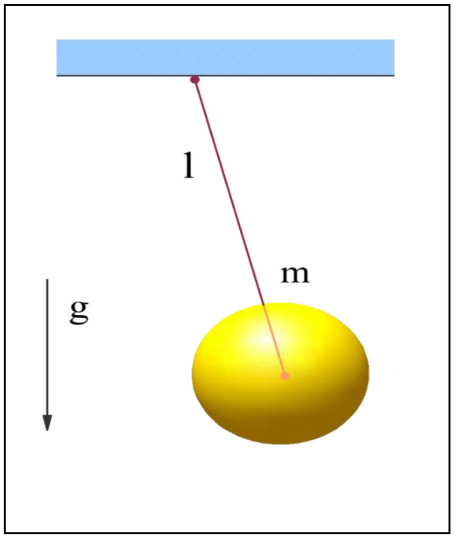
EXAMPLE

Consider that the time period (T) of a simple pendulum depends upon:

1. Mass of the bob (m),

2. Effective length of the pendulum (l), and

3. Acceleration due to gravity (g).



SOLUTION:

As the time period depends upon m, l and g. Let the relation between the quantities be:

$$T = K\, m^a\, l^b\, g^c$$

where K = dimensionless constant. a, b and c are the powers or exponents of mass, length and acceleration due to gravity respectively.

If the above relation is dimensionally correct then terms on both sides should have the same dimensions.

By substituting the dimensions of quantities and adding exponents using the rules –

$$[T] = [M]^a\, [L]^b\, [LT^{-2}]^c$$

or $[M^0L^0T^1] = [M^a\, L^{b+c}\, T^{-2c}]$

next using the principle of homogeneity, which states that the exponents of each base quantity on both sides of an equality should be the same

Equating the powers on same quantities of both the sides, we get

$a = 0$,

b + c =0

-2c = 1

Solving the equations for a, b, and c, we get

a = 0,

b = 1/2 and

c = – 1/2

So the required physical relation becomes

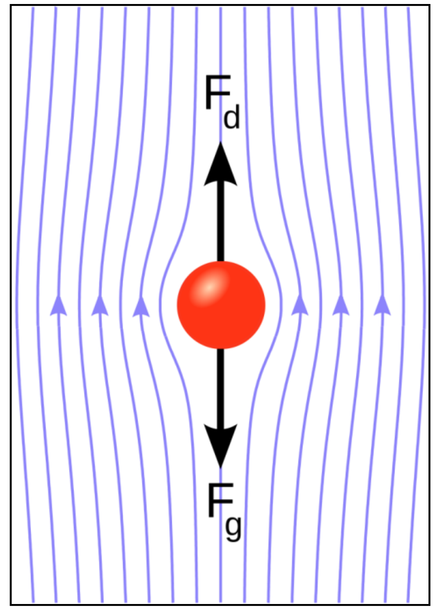
$$\mathbf{T = K\sqrt{\frac{l}{g}}}$$

This method tells how time period is related to m, l and g but it does not tell anything about the dimensionless constant K. Thus, we need to find out using other methods.

Experimentally, the value of dimensionless constant K is found 2π, so, the expression for time period is

$$\mathbf{T = 2\pi\sqrt{\frac{l}{g}}}$$

EXAMPLE



When a small sphere moves at low speed through a fluid, the viscous force *F*, opposing the motion, is found experimentally to depend on the radius *r*, the velocity of the sphere v and the viscosity *η* of the fluid.

SOLUTION:

$F = K\ \eta^a\ r^b\ v^c$

where *K* is dimensionless constant.

If the above relation is dimensionally correct,

$[M^1\ L^1\ T^{-2}] = [M^1\ \ L^{-1}\ T^{-1}]^a\ [M^0\ \ L^1\ T^0]^b\ [M^0\ \ L^1\ \ T^{-1}\ ]^c$

Combining the powers of M, L and T

$$[M^1\ L^1\ T^{-2}] = \left[M^a\ L^{-a+b+c}\ T^{-a-c}\right]$$

Equating the powers of M L T on both the sides

We get $a = 1;$

$-a + b + c = 1$

and $-a - c = -2$

Solving these for a, b, c we get $a = b = c = 1$

So, $F = K\ \eta\ r\ v$

Experimentally $K = 6\pi;$

So, $F = 6\ \pi\ \eta\ r\ v$

This is the famous Stoke's law about which you learn later in class 11.

9. LIMITATIONS OF DIMENSIONAL ANALYSIS

Although dimensional analysis has many applications it has some limitations too, which are:

a) If dimensions are given, physical quantity may not be unique as many physical quantities have same dimensions. For example if the dimensional formula of a physical quantity is $[\mathrm{M\ L^2 T^{-2}}]$ it may be work or energy or torque.

b) Numerical constant having no dimensions such as (1/2), 1 or π *etc*. cannot be deduced by the methods of dimensions.

c) The method of dimensions cannot be applied to derive formula if a physical quantity depends on more than 3 physical quantities.

10.SUMMARY

- The dimensions of base quantities and combination of these dimensions describe the nature of physical quantities.
- Dimensional analysis can be used to check the dimensional consistency of equations, deducing relations among the physical quantities, etc.
- A dimensionally consistent equation need not be actually an exact (correct) equation, but a dimensionally wrong or inconsistent equation must be wrong.
- Dimensional analysis is very useful in deducing relations among the interdependent physical quantities. However, dimensionless constants cannot be obtained by this method. The method of dimensions can only test the dimensional validity, but not the exact relationship between physical quantities in any equation. It does not distinguish between the physical quantities having same dimensions.